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Fuzzy Approaches to Multicriteria de Novo Programs

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Multicriteria de Novo programming is a promising tool for optimal system design. However, there exist no algorithms for solving a general multicriteria de Novo program. Only special cases have been discussed. This paper proposes a two-step fuzzy approach based on the ideal and negative ideal solutions. It is shown that this approach is very efficient and is applicable to the general multicriteria de Novo programming. The fuzzy version of this problem is also formulated and analyzed. Numerical examples are given to illustrate the approaches.

1. INTRODUCTION

Decision problems with multiple criteria have been studied by many researchers. Most of these investigations concentrated on the optimization of a given system, while de Novo programming deals with the design of an optimal system. In general, a de Novo program with multiple objectives can be formulated as [8]

$$
\begin{aligned}
&\max \quad Z_k = \sum_{j=1}^{n} C_{kj} X_j, && k = 1, 2, ..., l,\\
&\min \quad W_s = \sum_{j=1}^{n} C_{sj} X_j, && s = 1, 2, ..., r,\\
&\text{subject to:} && \qquad\qquad (\mathrm{P1})\\
&\sum_{j=1}^{n} a_{ij} X_j - b_i \leqslant 0, && i = 1, 2, ..., m,\\
&\sum_{j=1}^{m} p_i b_i = B\\
&X_j \geqslant 0, && j = 1, 2, ..., n,
\end{aligned}
$$

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where X_j, b_i are decision variables for projects and resources respectively, p_i, represents the price of resources i, and B is the total available budget. Note that $\sum_{i=1}^{m} p_i a_{ij} = A_j$ represents the unit cost of product j. Using A_j, problem (P1) can be reformulated as

$$\begin{aligned} &\max \quad Z_k = \sum_{j=1}^{n} C_{kj} X_j, && k = 1, 2, ..., l, \\ &\min \quad W_s = \sum_{j=1}^{n} C_{sj} X_j, && s = 1, 2, ..., r, \\ &\text{subject to:} && \qquad\qquad \text{(P2)} \\ &\sum_{j=1}^{n} A_j X_j = B \\ &X_j \geqslant 0, && j = 1, 2, ..., n. \end{aligned}$$

If we maximize each of the objectives $Z_1, Z_2, ..., Z_l$ or minimize each of the objectives $W_1, W_2, ..., W_r$ independently and subject to the given constraint, we obtain the ideal solution: $0^* = (Z_1^*, ..., Z_l^* W_1^*, ..., W_r^*)$. This ideal solution 0^* is generally infeasible. Thus, a nondominated feasible solution is needed.

In the special case where the number of decision variables is equal to the number of decision criteria, this nondominated feasible solution can be obtained by solving a set of linear algebraic equations [7]. However, there exist no algorithms to solve the general de Novo programming problem.

In this paper, the general de Novo programming problem is approached based on the fuzzy methodology. Furthermore, a fuzzy de Novo programming is also formulated and solved. This fuzzy approach is more flexible than the standard one in treating both the constraints and the objectives. Numerical examples are given to illustrate the approach.

2. Fuzzy Approach to de Novo Programming

Consider the general model of de Novo programming, (P1). Because only one budgetary constraint is involved, the optimal solutions with respect to each objective function can be obtained simply by finding

$\forall k = 1, 2, ..., l$:

$$j^* = \left\{ j \in (1, 2, ..., n) \mid \max_j \left(C_{kj} \Big/ \sum_{j=1}^{m} p_i a_{ij} \right) \right\},$$

then

$$X_{kj}^{*} = \begin{cases} B \Big/ \sum\limits_{j=1}^{m} p_i a_{ij}, & \text{for} \quad j = j^{*}, \\ 0, & \text{otherwise} \end{cases}$$

with

$$z_k^{*} = \sum_{j=1}^{n} C_{kj} X_{kj}^{*} \tag{1}$$

for objectives Z_k and

$\forall s = 1, 2, ..., r$:

$$j^{*} = \left\{ j \in (1, 2, ..., n) \,\middle|\, \min_j \left(C_{sj} \Big/ \sum_{j=1}^{m} p_i a_{ij} \right) \right\},$$

then

$$X_{sj}^{*} = \begin{cases} B \Big/ \sum\limits_{j=1}^{m} p_i a_{ij}, & \text{for} \quad j = j^{*}, \\ 0, & \text{otherwise,} \end{cases}$$

with

$$W_s^{*} = \sum_{j=1}^{n} C_{sj} X_{sj}^{*} \tag{2}$$

for objectives W_s. The so-called ideal solution in compromise programming is given by $0^{*} = (Z_1^{*}, Z_2^{*}, ..., Z_l^{*}, W_2^{*}, ..., W_r^{*})$. Obviously, this ideal solution is not a feasible solution for the current budget, but it shows the inherent goals for the multicriterion system design.

Similarly, we can define the negative ideal solution and denote it by $0^{-} = (Z_1^{-}, Z_2^{-}, ..., Z_l^{-};\ W_1^{-}, W_2^{-}, ..., W_r^{-})$. The negative ideal solution shows the worst possible performances of an admissible system design and Z_k^{-} and W_s^{-} are considered as the tolerance limits for each objective, respectively, and are obtained by:

$\forall k = 1, 2, ..., l$:

$$j^{-} = \left\{ j \in (1, 2, ..., n) \,\middle|\, \min_j \left(C_{kj} \Big/ \sum_{j=1}^{m} p_i a_{ij} \right) \right\},$$

then

$$X_{kj}^{-}=\begin{cases} B\Big/\sum_{j=1}^{m} p_i a_{ij}, & \text{for } j=j^{-}, \\ 0, & \text{otherwise,} \end{cases}$$

with

$$Z_k^{-}=\sum_{j=1}^{n} C_{kj} X_{kj}^{-} \tag{3}$$

for Z_k and

$\forall s=1, 2, ..., r.$

$$j^{-}=\left\{ j \in (1, 2, ..., n) \,\Big|\, \max_j \left(C_{sj} \Big/ \sum_{j=1}^{m} p_i a_{ij} \right) \right\},$$

then

$$X_{sj}^{-}=\begin{cases} B\Big/\sum_{j=1}^{m} p_i a_{ij}, & \text{for } j=j^{-} \\ 0, & \text{otherwise,} \end{cases}$$

with

$$W_s^{-}=\sum_{j=1}^{n} C_{sj} X_{sj}^{-} \tag{4}$$

for W_s. With reference to both ideal and negative ideal solutions, we redefine the required set of membership functions as follows:

$\forall k=1, 2, ..., l$:

$$\mu_{\tilde{Z}_k}(Z_k)=\begin{cases} 0 & Z_k \leqslant Z_k^{-} \\ \dfrac{Z_k - Z_k^{-}}{Z_k^{*} - Z_k^{-}}, & Z_k^{-} < Z_k \leqslant Z_k^{*} \\ 1 & Z_k > Z_k^{*} \end{cases} \tag{$\mu1$}$$

and

$\forall s=1, 2, ..., r$:

$$\mu_{\tilde{W}_s}(w_s)=\begin{cases} 1 & W_s \leqslant W_s^{*} \\ \dfrac{W_s^{-} - W_s}{W_s^{-} - W_s^{*}}, & W_s^{*} < W_s \leqslant W_s^{-} \\ 0 & W_s > W_s^{-}. \end{cases} \tag{$\mu2$}$$

These functions indicate the degree of satisfaction related to the selection reference points for each objective. We only need to maximize the overall satisfaction of the system design. Using these membership functions, problem (P1) is transformed to [9]:

$$
\begin{aligned}
&\max \quad \lambda \\
&\text{subject to:} \quad \lambda \leqslant \left(\sum_{j=1}^{n} C_{kj} X_j - Z_k^- \right) \Big/ (Z_k^* - Z_k^-), && k = 1, 2, \ldots, l, \\
&\qquad \lambda \leqslant \left(W_s^- - \sum_{j=1}^{n} a_{ij} X_j \right) \Big/ (W_s^- - W_s^*), && s = 1, 2, \ldots, r, \text{(P3)} \\
&\qquad \sum_{j=1}^{n} A_j X_j = B \\
&\qquad \lambda \in [0, 1], X_j \geqslant 0, \quad j = 1, 2, \ldots, n.
\end{aligned}
$$

Because the "min" operator used in (P3) represents a "fuzzy and" which is compensatory, the solution given by (P3) may not be an efficient solution. A nonefficient solution is less attractive to the decision maker. Several other operators have been suggested in the literature [3, 9, 10]. All of them are fairly limited either due to nonefficient solutions or due to the fact that the resulting models become nonlinear.

We propose a two-step approach to solve this problem. This is a fairly natural approach which guarantees a nondominated system design. First, solve problem (P3) with noncompnesatory operator "min" to obtain the optimal value of λ related to the logical "fuzzy and." Then, formulate the following new problem (P4) with fully compensatory operator "Σ" to obtain the optimal arithmetic mean of all the membership values restricted by $\lambda_i \geqslant \lambda$, $\forall i = 1, \ldots, l + r$:

$$
\begin{aligned}
&\max \quad \bar{\lambda} = \frac{1}{l+r} \sum_{i=1}^{1+r} \lambda_i \\
&\text{subject to:} \\
&\lambda \leqslant \lambda_k \leqslant \left(\sum_{j=1}^{n} C_{kj} X_j - Z_k^- \right) \Big/ (Z_k^* - Z_k^-), \qquad k = 1, 2, \ldots, l, \\
&\lambda \leqslant \lambda_s \leqslant \left(W_s^- - \sum_{j=1}^{n} C_{sj} X_j \right) \Big/ (W_s^- - W_s^*), \qquad s = 1, 2, \ldots, r, \qquad \text{(P4)} \\
&\sum_{j=1}^{n} A_j X_j = B, \\
&\lambda_k, \lambda_s \in [0, 1], X_j \geqslant 0, \qquad j = 1, 2, \ldots, n.
\end{aligned}
$$

In a broad sense, this arithmetic mean value $\bar{\lambda}$ represents the logical "fuzzy or." The solution obtained from (P4) is efficient and the proof is obvious. To illustrate the approach, consider the same numerical example given in Zeleney [7]:

EXAMPLE 1.

$$\begin{aligned}
\max \quad & Z_1 = 50x_1 + 100x_2 + 17.5x_3 \\
& Z_2 = 92x_1 + 75x_2 + 50x_3 \\
& Z_s = 25x_1 + 100x_2 + 75x_3
\end{aligned}$$

subject to:

$$\begin{aligned}
& 12x_1 + 17x_2 \leqslant b_1 \\
& 3x_1 + 9x_2 + 8x_3 \leqslant b_2 \\
& 10x_1 + 13x_2 + 15x_3 \leqslant b_3 \\
& 6x_1 + 16x_3 \leqslant b_4 \\
& 12x_2 + 7x_3 \leqslant b_5 \\
& 9.5x_1 + 9.5x_2 + 4x_3 \leqslant b_6 \\
& 0.75b_1 + 0.6b_2 + 0.35b_3 + 0.5b_4 + 1.15b_5 + 0.65b_6 = 4658.75,
\end{aligned}$$

which is equivalent to

$$\max \quad Z = (Z_1, Z_2, Z_3)$$

subject to:

$$23.475x_1 + 42.675x_2 + 28.7x_3 = 4658.75.$$

using equations (1)–(4) we obtained the following ideal and negative ideal solutions:

$$\begin{aligned}
Z_1^* &= 10916.813, && \text{for} \quad x_1 = 0,\ x_2 = 109.16813,\ x_3 = 0; \\
Z_2^* &= 18257.933, && \text{for} \quad x_1 = 198.4558,\ x_2 = 0,\ x_3 = 0; \\
Z_3^* &= 12174.433, && \text{for} \quad x_1 = 0,\ x_2 = 0,\ x_3 = 162.32578; \\
Z_1^- &= 2840.7012, && \text{for} \quad x_1 = 0,\ x_2 = 0,\ x_3 = 162.32578; \\
Z_2^- &= 8116.289, && \text{for} \quad x_1 = 0,\ x_2 = 0,\ x_3 = 162.32578; \\
Z_3^- &= 2729.2033, && \text{for} \quad x_1 = 109.16813,\ x_2 = 0,\ x_3 = 0.
\end{aligned}$$

Using the notation introduced, we have $0^* = (10916.813,\ 18257.933,\ 12174.433)$ and $0^- = (2840.7012,\ 8116.289,\ 2729.2033)$. Based on these reference points, we can obtain the following set of membership functions by using equations (μ_1) and (μ_2).

$$\mu_{\tilde{z}_1}(z_1) = (50x_1 + 100x_2 + 17.5x_3 - 2840.7012)/8076.1118,$$

$$\mu_{\tilde{z}_2}(z_2) = (92x_1 + 75x_2 + 50x_3 - 8116.289)/10141.644,$$

$$\mu_{\tilde{z}_3}(z_3) = (25x_1 + 100x_2 + 75x_3 - 2729.2033)/9445.2297.$$

The original problem can thus be further transformed into

$$\begin{aligned}
&\max \quad \lambda \\
&\text{subject to:} \\
&\lambda \leqslant \mu_{\tilde{z}_i}(z_i), \qquad i = 1, 2, 3, \\
&23.475x_1 + 42.675x_2 + 28.7x_3 = 4658.75, \\
&\lambda \in [0, 1], \qquad x_1, x_2, x_3 \geqslant 0,
\end{aligned}$$

which yields $\lambda = 0.56$ and $x = (111.43,\ 7.60,\ 59.89)$. Thus the optimal system design is: $Z = (7379.13,\ 13815.46,\ 8037.02)$ with $b = (1466.35,\ 881.80,\ 2111.44,\ 1626.81,\ 510.42,\ 1370.34)$. It is easy to show that this solution is efficient.

To illustrate the fact that the operator "min" is noncompensatory, let us consider the following example:

EXAMPLE 2.

$$\begin{aligned}
\max \quad & Z_1 = 2x_1 - 5x_2 + 7x_3 = x_4 \\
& Z_2 = 4x_1 + x_2 + 3x_3 + 11x_4 \\
& Z_3 = 9x_1 + 3x_2 + x_3 + 2x_4 \\
\min \quad & W_1 = 1.5x_1 + 2x_2 + 0.3x_3 + 3x_4 \\
& W_2 = 0.5x_1 + x_2 + 0.73x_3 + 2x_4
\end{aligned}$$

subject to:

$$3x_1 + 4.5x_2 + 1.5x_3 + 7.5x_4 = 150$$

$$x_1, x_2, x_3, x_4 \geqslant 0.$$

The ideal solution is $0^* = (700, 300, 450; 30, 250$ and the negative ideal solution is $0^- = (20, 33.33, 40; 75, 70)$. The corresponding program to (P3) is

$$\max \quad \lambda$$

subject to:

$$\lambda \leqslant \frac{1}{680}(2x_1 + 5x_2 + 7x_3 + x_4 - 20),$$

$$\lambda \leqslant \frac{1}{266.67}(4x_1 + x_2 + 3x_3 + 11x_4 - 33.33),$$

$$\lambda \leqslant \frac{1}{410}(0x_1 + 3x_2 + x_3 + 2x_4 - 40),$$

$$\lambda \leqslant \frac{1}{45}(75 - 1.5x_1 - 2x_2 - 0.3x_3 - 3x_4),$$

$$\lambda \leqslant \frac{1}{45}(70 - 0.5x_1 - x_2 - 0.7x_3 - 2x_4),$$

$$3x_1 + 4.5x_2 + 1.5x_2 + 7.5x_2 = 150,$$

$$\lambda \in [0, 1], x_1, x_2, x_3, x_4 \geqslant 0.$$

Using Lindo or Macrosolve computer programs, we obtained the following solution: $\lambda = \min(0.55, 0.74, 0.5, 0.5, 0.5) = 0.5) = 0.5$ and $x = (20.71, 3.51, 48.05, 0)$. Thus the system performance is $Z = (395.32, 230.52, 245.00; 52.5, 47.5)$. On the other hand, the corresponding program to (P4) is

$$\max \quad \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5$$

subject to:

$$0.5 \leqslant \lambda_1 \leqslant \frac{1}{680}(2x_1 + 5x_2 + 7x_3 + x_4 - 20),$$

$$0.5 \leqslant \lambda_2 \leqslant \frac{1}{266.67}(4x_1 + x_2 + 3x_3 + 11x_4 - 33.33),$$

$$0.5 \leqslant \lambda_3 \leqslant \frac{1}{410}(9x_1 + 3x_2 + x_3 + 2x_4 - 40),$$

$$0.5 \leqslant \lambda_4 \leqslant \frac{1}{45}(75 - 1.5x_1 - 2x_1 - 0.3x_3 - 3x_4),$$

$$0.5 \leqslant \lambda_5 \leqslant \frac{1}{45}(70 - 0.5x_1 - x_2 - 0.7x_3 - 2x_4),$$

$$3x_1 + 4.5x_2 + 1.5x_3 + 7.5x_4 = 150$$

$$\lambda \in [0, 1], x_1, x_2, x_3, x_4 \geqslant 0,$$

which yields $\lambda = (0.56, 0.81, 0.57; 0.50, 0.50)$ and $X = (25.00, 0, 50.00, 0)$.

The optimal system performance is $Z = (400.00, 250.00, 275.00; 52.50, 47.50)$.

Comparing the above results, it is clear that program (P3) gives only a nonefficient solution in which the system performance is either equal to or dominated by the ones given by program (P4).

3. Fuzzy de Novo Programming [2]

Standard de Novo pogramming, like other traditional linear or nonlinear multipbjective programs, requires a precisely known data structure. In the real world, however, it is often difficult to determine the coefficients of the variables precisely. To make standard de Novo programming more flexible, we introduce a fuzzy environment into the general model of the Novo programming such that

$$\begin{aligned}
&\max \quad \tilde{Z}_k = \sum_{j=1}^{n} \tilde{C}_{kj} X_j, \qquad k = 1, 2, ..., l, \\
&\min \quad \tilde{W}_s = \sum_{j=1}^{n} \tilde{C}_{sj} X_j, \qquad s = 1, 2, ..., r, \\
&\text{subject to} \quad \sum_{j=1}^{n} \tilde{a}_{ij} X_j - b_i \leqslant 0, \qquad i = 1, 2, ..., m, \qquad \text{(P5)} \\
&\quad \sum_{i=1}^{m} \tilde{p}_i b_i = B \\
&\quad\quad X_j \geqslant 0, \qquad j = 1, 2, ..., n,
\end{aligned}$$

where parameters $\tilde{C}_{kj}$, $\tilde{a}_{ij}$, $\tilde{P}_i$, $\tilde{B}$ are fuzzy numbers on R characterized by their membership functions $\mu_{\tilde{C}_{kj}}$, $\mu_{\tilde{a}_{ij}}$, $\mu_{\tilde{P}_i}$, $\mu_{\tilde{B}}$, respectively.

Let $(X)_\alpha$ be a solution of (P5), where $\alpha \in [0, 1]$ represents the degree of possibility to which the solution satisfies the problem. We have

$$\alpha = \min \left\{ \text{Poss}(\tilde{Z}_k) \mid \forall k = 1, 2, ..., l, \ \text{Poss}(\tilde{W}_s) \mid \forall s = 1, 2, ..., r, \right.$$

$$\left. \text{Poss}\left(\sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{P}_i \tilde{a}_{ij} X_j = \tilde{B} \right) \right\},$$

where Poss denotes possibility, and

$\forall k = 1, 2, ..., l$:

$$\text{Poss}(\tilde{Z}_k) = \min \left\{ \mu_{\tilde{C}_{kj}}(C_{kj}) \mid \forall j = 1, 2, ..., n, \ Z_k = \sum_j C_{kj} X_j \right\},$$

$\forall s = 1, 2, ..., r$:

$$\text{Poss}(\tilde{W}_s) = \min\left\{\mu_{\tilde{C}_{sj}}(C_{sj}) \mid \forall j = 1, 2, ..., n,\ W_s = \sum_j C_{sj} X_j\right\},$$

and

$$\text{Poss}\left(\sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{P}_i \tilde{a}_{ij} X_j = \tilde{B}\right)$$
$$= \sup\min\left\{\mu_{\tilde{p}_i}(p_i),\ \mu_{\tilde{a}_{ij}}(a_{ij}),\ \mu_{\tilde{B}}(B)\right.$$
$$\left. \mid \forall i = 1, 2, ..., m,\ j = 1, 2, ..., n,\ \sum_i \sum_j P_i a_{ij} X_j = B\right\}.$$

As shown in [1, 2], if the decision maker can specify a type of interval $[W^0, W^1)$ for parameters $\tilde{C}_{kj}$ and $\tilde{B}$, and a type of interval $(Q^1, Q^2]$ for parameters $\tilde{a}_{ij}$ and $\tilde{P}_i$, where superscript 0 means "risk free" and superscript 1 means "impossible," then referring to the definition of α, the original formulation of program (P5) can be transformed to

$$\begin{aligned} &\max \quad (\tilde{Z}_k)_\alpha = \sum_{j=1}^{n} \mu^{-1}_{\tilde{C}_{kj}}(\alpha)\, X_j, && k = 1, 2, ..., l, \\ &\min \quad (\tilde{W}_s)_\alpha = \sum_{j=1}^{n} \mu^{-1}_{\tilde{C}_{sj}}(\alpha)\, X_j, && s = 1, 2, ..., r, \\ &\text{subject to} \quad \sum_{i=1}^{m} \sum_{j=1}^{n} \mu^{-1}_{\tilde{p}_i}(\alpha)\, X_j = \mu^{-1}_{\tilde{B}}(\alpha) && \\ &\qquad \alpha \in [0, 1],\ X_j \geqslant 0, && j = 1, 2, ..., n, \end{aligned} \tag{P6}$$

where $(\tilde{Z}_k)_\alpha$, $(\tilde{W}_s)_\alpha$ are the α-cuts of $\tilde{Z}_k$ and $\tilde{W}_s$, defined by

$$(\tilde{Z}_k)_\alpha = \{Z_k \in R \mid \mu_{\tilde{Z}_k}(Z_k) \geqslant \alpha\}, \qquad k = 1, 2, ..., l,$$
$$(\tilde{W}_s)_\alpha = \{W_s \in R \mid \mu_{\tilde{W}_s}(W_s) \geqslant \alpha\}, \qquad s = 1, 2, ..., r,$$

and

$$\begin{aligned} &\mu^{-1}_{\tilde{C}_{kj}}(\alpha) = C^0_{kj} - \alpha(C^1_{kj} - C^0_{kj}), && k = 1, 2, ..., l,\ j = 1, 2, ..., n, \\ &\mu^{-1}_{\tilde{C}_{sj}}(\alpha) = C^1_{sj} + \alpha(C^0_{sj} - C^1_{sj}), && s = 1, 2, ..., r,\ j = 1, 2, ..., n, \\ &\mu^{-1}_{\tilde{P}_i}(\alpha) = P^1_i + \alpha(P^0_i - P^1_i), && i = 1, 2, ..., m \\ &\mu^{-1}_{\tilde{a}_{ij}}(\alpha) = a^1_{ij} + \alpha(a^0_{ij} - a^1_{ij}), && i = 1, ..., m,\ j = 1, 2, ..., n \\ &\mu^{-1}_{\tilde{B}}(\alpha) = b^0 - \alpha(b^1 - b^0). && \end{aligned}$$

For a given $\alpha \in \{0, 1]$, the ideal solution 0* and the negative ideal solution 0 can be found from the following procedures:

$\forall k = 1, ..., l$:

$$j_k^* = \left\{ j \in (1, 2, ..., n) \,\middle|\, \max\left(\mu_{\tilde{C}_{kj}}^{-1}(\alpha) \middle/ \sum_{i=1}^{m} \mu_{\tilde{P}_i}^{-1}(\alpha)\, \mu_{\tilde{a}_{ij}}^{-1}(\alpha)\right)\right\},$$

$$X_{kj}^* = \begin{cases} \mu_{\tilde{B}}^{-1}(\alpha) \middle/ \sum_{i=1}^{m} \mu_{\tilde{P}_i}^{-1}(\alpha)\, \mu_{\tilde{a}_{ij}}^{-1}(\alpha), & \text{for } j = j_k^{**}, \\ 0, & \text{otherwise,} \end{cases}$$

$$(\tilde{Z}_k)_\alpha^* = \sum_{j=1}^{n} \mu_{\tilde{C}_{kj}}^{-1}(\alpha)\, X_{kj}^*$$

and

$$j_k^- = \left\{ j \in (1, 2, ..., n) \,\middle|\, \min\left(\mu_{\tilde{C}_{kj}}^{-1}(\alpha) \middle/ \sum_{i=1}^{m} \mu_{\tilde{P}_i}^{-1}(\alpha)\, \mu_{\tilde{a}_{ij}}^{-1}(\alpha)\right)\right\},$$

$$X_{kj}^- = \begin{cases} \mu_{\tilde{B}}^{-1}(\alpha) \middle/ \sum_{i=1}^{m} \mu_{\tilde{p}_i}^{-1}(\alpha)\, \mu_{\tilde{a}_{ij}}^{-1}(\alpha), & \text{for } j = j_k^-, \\ 0, & \text{otherwise,} \end{cases}$$

$$(\tilde{Z}_k)_\alpha^- = \sum_{j=1}^{n} \mu_{\tilde{C}_{kj}}^{-1}(\alpha)\, X_{kj}^-,$$

$\forall s = 1, 2, ..., r$:

$$j_s^* = \left\{ j \in (1, 2, ..., n) \,\middle|\, \min\left(\mu_{\tilde{C}_{sj}}^{-1}(\alpha) \middle/ \sum_{I=1}^{r} \mu_{\tilde{P}_I}^{-1}(\alpha)\, \mu_{\tilde{C}_{Ij}}(\alpha)\right)\right\},$$

$$X_{sj}^* = \begin{cases} \mu_{\tilde{B}}^{-1}(\alpha) \middle/ \sum_{I=1}^{r} \mu_{\tilde{p}_I}^{-1}(\alpha)\, \mu_{\tilde{C}_{Ij}}^{-1}(\alpha), & \text{for } j = j_s^*, \\ 0, & \text{otherwise,} \end{cases}$$

$$(\tilde{W}_s)_\alpha^* = \sum_{j=1}^{n} \mu_{\tilde{C}_{sj}}^{-1}(\alpha)\, X_{sj}^*$$

and

$$j_s^- = \left\{ j \in (1, 2, ..., n) \,\middle|\, \max \left(\mu_{\tilde{C}_{sj}}^{-1}(\alpha) \Big/ \sum_{l=1}^{s} \mu_{\tilde{P}_l}^{-1}(\alpha)\, \mu_{\tilde{C}_{lj}}^{-1}(\alpha) \right) \right\},$$

$$X_{sj}^- = \begin{cases} \mu_{\tilde{B}}^{-1}(\alpha) \Big/ \sum_{l=1}^{r} \mu_{\tilde{P}_l}^{-1}(\alpha)\, \mu_{\tilde{C}_{lj}}^{-1}(\alpha), & \text{for } j = j_s^-, \\ 0, & \text{otherwise,} \end{cases}$$

$$(\tilde{W}_s)_\alpha^- = \sum_{j=1}^{n} \mu_{\tilde{C}_{sj}}^{-1}(\alpha)\, X_{sj}^- .$$

As has been discussed previously, problem (P6) can be solved in two steps. The first step is to solve the problem:

$$\max \quad (\tilde{\lambda})_\alpha$$

subject to

$$(\tilde{\lambda})_\alpha \leqslant \left(\sum_{j=1}^{n} \mu_{\tilde{C}_{kj}}^{-1}(\alpha)\, X_j - (\tilde{Z}_k)_\alpha^- \right) \Big/ ((\tilde{Z}_k)_\alpha^* - (Z_k)_\alpha^-), \qquad k = 1, 2, ..., l$$

$$(\tilde{\lambda})_\alpha \leqslant \left((\tilde{W}_s)_\alpha^- - \sum_{j=1}^{n} \mu_{\tilde{C}_{sj}}^{-1}(\alpha)\, X_j \right) \Big/ ((\tilde{W}_s)_\alpha^- - (\tilde{W}_s)_\alpha^*), \qquad s = 1, 2, ..., r, \quad \text{(P7)}$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} \mu_{\tilde{P}_i}^{-1}(\alpha)\, \mu_{\tilde{a}_{ij}}^{-1}(\alpha)\, X_j = \mu_{\tilde{B}}^{-1}(\alpha),$$

$$(\tilde{\lambda})_\alpha \in [0, 1], X_j \geqslant 0, \qquad j = 1, 2, ..., n.$$

The second step is to solve the problem

$$\max \quad \frac{1}{l+r} \sum_{l=1}^{l+r} (\tilde{\lambda}_l)\alpha$$

subject to:

$$(\tilde{\lambda})_\alpha \leqslant (\tilde{\lambda}_k)_\alpha = \left(\sum_{j=1}^{n} \mu_{\tilde{c}_{kj}}^{-1}(\alpha)\, X_j - ((Z_k)_\alpha^- \right) \Big/ ((Z_k)_\alpha^* - (Z_k)_\alpha^-), \qquad k = 1, ..., l$$

$$(\tilde{\lambda})_\alpha \leqslant (\tilde{\lambda}_s)_\alpha = \left((\tilde{W}_s)_\alpha^- - \sum_{s=1}^{n} \mu_{\tilde{c}_{sj}}^{-1}(\alpha)\, X_j \right) \Big/ ((\tilde{W}_s)_\alpha^- - (\tilde{W}_s)_\alpha^*), \qquad s = 1, ..., r,$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} \mu_{\tilde{p}_i}^{-1}(\alpha)\, \mu_{\tilde{a}_{ij}}^{-1}(\alpha)\, X_j = \mu_{\tilde{B}}^{-1}(\alpha), \qquad \text{(P8)}$$

$$(\tilde{\lambda}_k)_\alpha, (\tilde{\lambda}_s)_\alpha \in [0, 1], X_j \geqslant 0, \qquad j = 1, ..., n.$$

The advantage of this approach is that the decision maker is allowed to participate in the decision process by choosing his appropriate membership grade based on the risk factor he is willing to take. The following example illustrates the approach.

EXAMPLE 3.

$$\begin{aligned}
\max \quad & \tilde{Z}_1 = [2^0, 5^1)\, X_1 + 12X_2, \\
& \tilde{Z}_2 = 3X_1 + [1^0, 3^1)\, X_2, \\
\min \quad & \tilde{b}_1 = X_1 + (1^1, 4^0]\, X_2, \\
& \tilde{b}_2 = 2X_1 + (2^1, 3^0]\, X_2, \\
\text{subject to} \quad & ((0.5^1, 2^0] + 2)\, X_1 + ((0.5^1, 2^0], (1^1, 4^0) + (2^1, 3^0])\, X_2 \\
& \quad = [200^0, 250^1),\ X_1, X_2, X_3 \geqslant 0.
\end{aligned}$$

The parameters can be expressed by means of their membership functions for a given α as:

$$\begin{array}{llll}
\mu^{-1}_{\tilde{c}_{11}}(\alpha) = 5 - 3\alpha, & \mu^{-1}_{\tilde{c}_{12}}(\alpha) = 12, & \mu^{-1}_{\tilde{c}_{21}}(\alpha) = 4, & \mu^{-1}_{\tilde{c}_{22}}(\alpha) = 4 - 2\alpha \\
\mu^{-1}_{\tilde{a}_{11}}(\alpha) = 1, & \mu^{-1}_{\tilde{a}_{12}}(\alpha) = 1 + 3\alpha, & \mu^{-1}_{\tilde{a}_{21}}(\alpha) = 2, & \mu^{-1}_{\tilde{a}_{22}}(\alpha) = 2 + \alpha, \\
\mu^{-1}_{\tilde{P}_1}(\alpha) = 0.5 + 1.5\alpha, & \mu^{-1}_{\tilde{P}_2}(\alpha) = 1, & \mu^{-1}_{\tilde{B}}(\alpha) \ = 250 - 50\alpha. &
\end{array}$$

Then the problem can be rewritten as

$$\begin{aligned}
\max \quad & (\tilde{Z}_1)_\alpha = (5 - 3\alpha)\, X_1 + 12X_2 \\
& (\tilde{Z}_2)_\alpha = 4X_1 + (3 - 2\alpha)\, X_2, \\
\min \quad & (\tilde{W}_1)_\alpha = X_1 + (1 + 3\alpha)\, X_2 \\
& (\tilde{W}_2)_\alpha = 2X_1 + (2 + \alpha)\, X_2 \\
\text{subject to:} \quad & (2.5 + 1.5\alpha)\, X_1 + (2.5 + 4\alpha + 4.5\alpha^2)\, X_2 = 250 - 50\alpha \\
& \alpha \in [0, 1],\ X_1, X_2 \geqslant 0.
\end{aligned}$$

To illustrate the approach, let $\alpha = 0.8$, then Example 3 is reduced to a standard de Novo problem. The optimal solution becomes: $\lambda = 0.50$ and $X = (28.38, 12.24)$, which leads to the optimal design: $\{\tilde{Z}, \tilde{W}, X, \tilde{B}\}_{\alpha = 0.8} = \{(220.7, 130.7), (70.0, 91.0), (28.38, 12.24), 210\}$. In the same way, we can also obtain the optimal system designs for $\alpha = 0.1$, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0 and this is shown in Fig. 1.

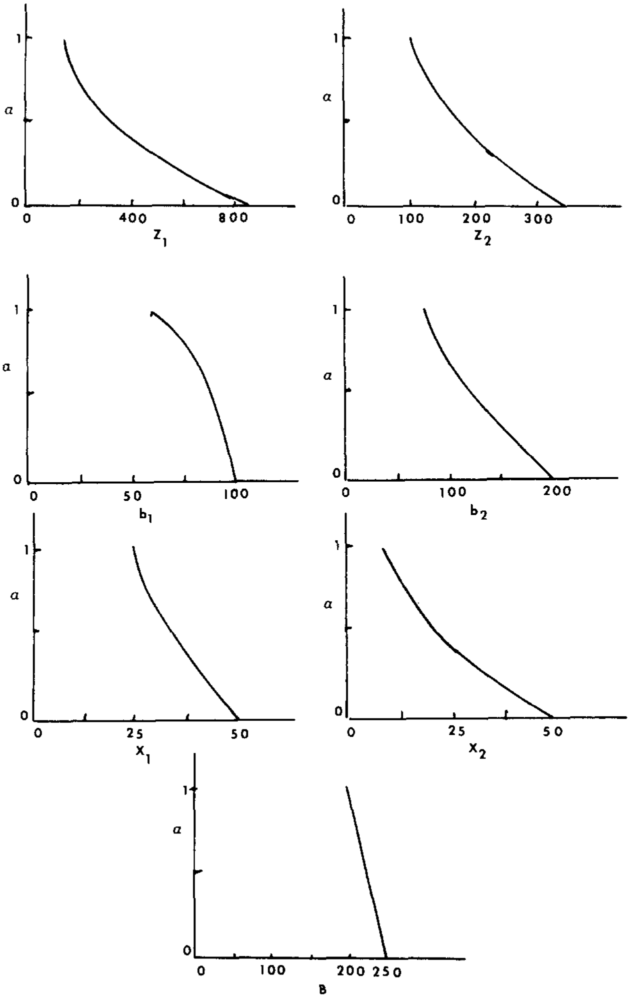

FIG. 1. System designs for different values of α.

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